

10th Asian Pacific Mathematics Olympiad

March 1998

Time allowed: 4 hours.

No calculators to be used.

Each question is worth 7 points.

1. Let F be the set of all n -tuples (A_1, A_2, \dots, A_n) where each $A_i, i = 1, 2, \dots, n$ is a subset of $\{1, 2, \dots, 1998\}$. Let $|A|$ denote the number of elements of the set A .

Find the number $\sum_{(A_1, A_2, \dots, A_n)} |A_1 \cup A_2 \cup \dots \cup A_n|$.

2. Show that for any positive integers a and b , $(36a+b)(a+36b)$ cannot be a power of 2.

3. Let a, b, c be positive real numbers. Prove that $\left(1 + \frac{a}{b}\right)\left(1 + \frac{b}{c}\right)\left(1 + \frac{c}{a}\right) \geq 2\left(1 + \frac{a+b+c}{\sqrt[3]{abc}}\right)$.

4. Let ABC be a triangle and D the foot of the altitude from A . Let E and F be on a line through D such that AE is perpendicular to BE , AF is perpendicular to CF , and E and F are different from D . Let M and N be the midpoints of the line segments BC and EF , respectively. Prove that AN is perpendicular to NM .

5. Determine the largest of all integers n with the property that n is divisible by all positive integers that are less than $\sqrt[3]{n}$.

END OF PAPER