XXXVIII Asian Pacific Mathematics Olympiad

March, 2026

Time allowed: 4 hours — *Each problem is worth 7 points*

The contest problems are to be kept confidential until they are posted on the official APMO website `http://apmo-official.org`*. Please do not disclose nor discuss the problems online until that date. The use of calculators is not allowed.*

Problem 1. Find all quadruples of positive integers (a, p, m, o) with $o \geq 2$ such that

$$a! + p! = m^o + 26.$$

Problem 2. Let $n \geq 2$ be an integer. Let $A_1, A_2, \cdots, A_{2^n}$ be the 2^n subsets of an n-element set in some order. Prove that

$$|A_1 \cap A_2| + |A_2 \cap A_3| + \cdots + |A_{2^n-1} \cap A_{2^n}| + |A_{2^n} \cap A_1| \geq 2^{n-2}.$$

Problem 3. Let $\mathbb{R}_+$ denote the set of positive real numbers. Determine all the functions $f : \mathbb{R}_+ \to \mathbb{R}$ such that for all $x, y, z \in \mathbb{R}_+$,

$$|x - y| < |y - z| \text{ if and only if } |f(x) - f(y)| < |f(y) - f(z)|.$$

Problem 4. Let $ABCD$ be a quadrilateral with an incircle ω of centre I. The diagonals AC and BD intersect at E. Let J be the incentre of triangle ABD. The extension of the ray EJ intersects ω at P. Prove that $PI \perp BD$.

Problem 5. Let n be a positive integer. There exist $n(n+1)$ rooms arranged in a $(n+1) \times n$ grid. For each two adjacent rooms, there is a door between them. Find the number of ways to choose a subset of doors and lock them such that there are two rooms S and G satisfying the following properties:

(i) S is in the first row and G is in the $(n+1)$-th row.

(ii) One can reach G from S only through unlocked doors.